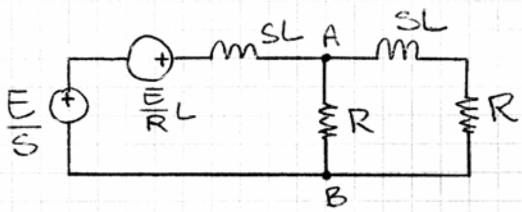
ESERCIZIO 1

condizioni iniziali : $i_{L1}(t<0) = \frac{E}{R}$ $\quad i_{L2}(t<0) = 0$

utilizzando laplace :



$E = 10V$

$R = 0{,}1 \cdot (10+k)\ \Omega$

$L = 0{,}1\ H$

$$V_{AB} = \frac{\frac{E/s + EL/R}{sL}}{\frac{1}{sL} + \frac{1}{R} + \frac{1}{R+sL}} = \frac{\frac{E/s + EL/R}{\cancel{sL}}}{\frac{R\cdot(R+sL) + sL\cdot(R+sL) + sRL}{\cancel{sL}R\,(R+sL)}} =$$

$$= \frac{1/s \cdot (E + s\,EL/R)\cdot R\cdot(R+sL)}{R^2 + \underline{sRL} + \underline{sRL} + s^2L^2 + \underline{sRL}} = \frac{1}{s}\cdot\frac{\frac{E}{\cancel{R}}\cdot(R+sL)\cdot\cancel{R}\cdot(R+sL)}{s^2L^2 + 3sRL + R^2} =$$

$$= \frac{E\cdot(R+sL)^2}{sL^2\cdot\left(s^2 + 3s\,R/L + R^2/L^2\right)}$$

le radici del denominatore sono:

$$s_{1,2} = -\frac{3}{2}\frac{R}{L} \pm \sqrt{\left(\frac{3}{2}\frac{R}{L}\right)^2 - \left(\frac{R}{L}\right)^2} = -\frac{3}{2}\frac{R}{L} \pm \sqrt{\frac{5}{4}\frac{R^2}{L^2}} = \begin{cases} s_1 = \frac{-3-\sqrt{5}}{2}\frac{R}{L} \\ s_2 = \frac{-3+\sqrt{5}}{2}\frac{R}{L} \end{cases}$$

scomposizione in fratti semplici :

$$V_{AB} = \frac{A}{s} + \frac{B}{s-s_1} + \frac{C}{s-s_2}$$

$A = \left[s\cdot V_{AB}(s)\right]\big|_{s=0}$

$B = \left[(s-s_1)\ V_{AB}(s)\right]_{s=s_1}$

$C = \left[(s-s_2)\cdot V_{AB}(s)\right]_{s=s_2}$

quindi:

$$A = \frac{ER^2}{L^2\cdot R^2/L^2} = E$$

$$B = \ldots = \frac{E}{\sqrt{5}}$$

$$C = \ldots = \frac{-E}{\sqrt{5}}$$

$$\Longrightarrow\ V_{AB}(s) = \frac{E}{s} + \frac{E/\sqrt{5}}{s + \frac{3+\sqrt{5}}{2}\frac{R}{L}} - \frac{E/\sqrt{5}}{s + \frac{3-\sqrt{5}}{2}\frac{R}{L}}$$

dopo avere antitrasformato:

$$V_{AB}(t) = 10 + \frac{10}{\sqrt{5}} \cdot e^{-s_1 t} - \frac{10}{\sqrt{5}} e^{-s_2 t}$$

$$V_{AB}(t) = 10 + \frac{10}{\sqrt{5}} \cdot \left(e^{-s_1 t} - e^{-s_2 t}\right)$$

dove:

$$s_1 = \frac{-3+\sqrt{5}}{2} \frac{R}{L} = -3{,}82 \cdot R$$

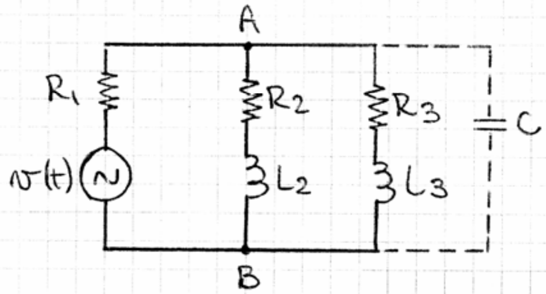
$k = 0 \rightarrow s_1 = -3{,}82$
$k = 1 \rightarrow s_1 = -4{,}20$
$k = 2 \rightarrow s_1 = -4{,}58$
$k = 3 \rightarrow s_1 = -4{,}97$
$k = 4 \rightarrow s = -5{,}35$
$k = 5 \rightarrow s_1 = -5{,}73$
$k = 6 \rightarrow s_1 = -6{,}11$
$k = 7 \rightarrow s_1 = -6{,}49$
$k = 8 \rightarrow s_1 = -6{,}88$
$k = 9 \rightarrow s_1 = -7{,}26$

$$s_2 = \frac{-3-\sqrt{5}}{2} \frac{R}{L} = -26{,}2 \cdot R$$

$k = 0 \rightarrow s_2 = -26{,}2$
$k = 1 \rightarrow s_2 = -28{,}8$
$k = 2 \rightarrow s_2 = -31{,}4$
$k = 3 \rightarrow s_2 = -34{,}0$
$k = 4 \rightarrow s_2 = -36{,}7$
$k = 5 \rightarrow s_2 = -39{,}3$
$k = 6 \rightarrow s_2 = -41{,}9$
$k = 7 \rightarrow s_2 = -44{,}5$
$k = 8 \rightarrow s_2 = -47{,}1$
$k = 9 \rightarrow s_2 = -49{,}7$

ESERCIZIO 2

Svolto per $K = 0$



$R_1 = 1\ \Omega$

$R_2 = R_3 = 10\ \Omega$

$L_2 = 20\ mH$

$L_3 = (10+K)\ mH$

$v(t) = 220\sqrt{2}\cos(314t)\quad [V]$

$\bar{Z}_2 = R_2 + X_{L2} = 10 + j6{,}3$ $\qquad$ $\bar{Z}_3 = R_3 + X_{L3} = 10 + j3{,}1$

$$\bar{Z}_{eq} = \bar{Z}_1 + \frac{Z_2 \cdot Z_3}{Z_2 + Z_3} = 1 + \frac{(10+j6{,}3)\cdot(10+j3{,}1)}{20+j9{,}4} =$$

$$= \frac{20+j9{,}4+100-19{,}5+63j+31j}{20+j9{,}4} = \frac{100{,}5+103{,}4j}{20+j9{,}4}\cdot\frac{20-j9{,}4}{20-j9{,}4} =$$

$$= \frac{2010+972+2068j-945j}{400+88{,}4} = 6{,}1+j\,2{,}3$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}_{eq}} = \frac{220}{6{,}1+j2{,}3} = \frac{6{,}1-j2{,}3}{42{,}5}\cdot 220 = 31{,}6 - j\,11{,}9 \quad [A]$$

$$i(t) = \sqrt{31{,}6^2+11{,}9^2}\,\sqrt{2}\cdot\cos\left(314t - \arctan\left(-\frac{11{,}9}{31{,}6}\right)\right)$$

$$i(t) = 33{,}8\sqrt{2}\cos(314t - 20°)$$

$$p(t) = v(t)\cdot i(t) = 220\sqrt{2}\cos(314t)\cdot 33{,}8\sqrt{2}\cos(314t-20°)$$

$$P = V\cdot I\cos\varphi = 220\cdot 33{,}8\cos 20° = 6987\quad [W]$$

$$Q = V\cdot I\,\mathrm{sen}\,\varphi = 220\cdot 33{,}8\,\mathrm{sen}\,20° = 2543\quad [VAR]$$

Rifasamento: supponendo trascurabile la caduta su R_1 si ha:

$v(t) = v_{AB}(t)$

$I_C = j\,11{,}9$ $\qquad$ $V = \frac{1}{\omega C} I_C$

$$C = \frac{I_C}{\omega V} = \frac{11{,}9}{314\cdot 220} = 172\ \mu F$$