



# **Formulari e Tabelle**

# Trasformata di Fourier: proprietà

## Fourier Transforms

### Definitions

Transform

$$V(f) = \mathcal{F}[v(t)] = \int_{-\infty}^{\infty} v(t) e^{-j2\pi ft} dt$$

Inverse transform

$$v(t) = \mathcal{F}^{-1}[V(f)] = \int_{-\infty}^{\infty} V(f) e^{j2\pi ft} df$$

### Integral theorem

$$\int_{-\infty}^{\infty} v(t) w^*(t) dt = \int_{-\infty}^{\infty} V(f) W^*(f) df$$

### Theorems

Operation	Function	Transform
Superposition	$a_1 v_1(t) + a_2 v_2(t)$	$a_1 V_1(f) + a_2 V_2(f)$
Time delay	$v(t - t_d)$	$V(f) e^{-j\omega t_d}$
Scale change	$v(\alpha t)$	$\frac{1}{ \alpha } V\left(\frac{f}{\alpha}\right)$
Conjugation	$v^*(t)$	$V^*(-f)$
Duality	$V(t)$	$v(-f)$
Frequent translation	$v(t) e^{j\omega_c t}$	$V(f - f_c)$
Modulation	$v(t) \cos(\omega_c t + \phi)$	$\frac{1}{2} [V(f - f_c) e^{j\phi} + V(f + f_c) e^{-j\phi}]$
Differentiation	$\frac{d^n v(t)}{dt^n}$	$(j2\pi f)^n V(f)$
Integration	$\int_{-\infty}^t v(\lambda) d\lambda$	$\frac{1}{j2\pi f} V(f) + \frac{1}{2} V(0) \delta(f)$
Convolution	$v * w(t)$	$V(f) W(f)$
Multiplication	$v(t) w(t)$	$V * W(f)$
Multiplication by $t^n$	$t^n v(t)$	$(-j2\pi)^{-n} \frac{d^n V(f)}{df^n}$

# Trasformate notevoli

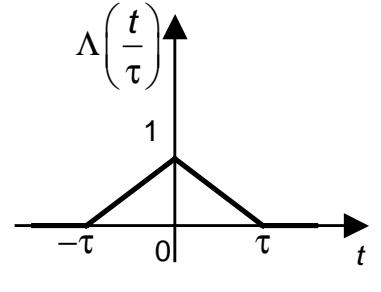
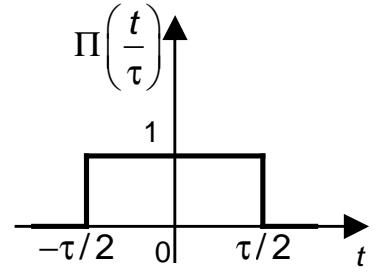
## Transforms

Function	$v(t)$	$V(f)$
Rectangular	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} f\tau$
Triangular	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 f\tau$
Gaussian	$e^{-\pi(bt)^2}$	$(1/b) e^{-\pi(f/b)^2}$
Causal exponential	$e^{-bt} u(t)$	$\frac{1}{b + j2\pi f}$
Symmetric exponential	$e^{-b t }$	$\frac{2b}{b^2 + (2\pi f)^2}$
Sinc	$\operatorname{sinc} 2Wt$	$\frac{1}{2W} \Pi\left(\frac{f}{2W}\right)$
Sinc squared	$\operatorname{sinc}^2 2Wt$	$\frac{1}{2W} \Lambda\left(\frac{f}{2W}\right)$
Constant	1	$\delta(f)$
Phasor	$e^{j(\omega_c t + \phi)}$	$e^{j\phi} \delta(f - f_c)$
Sinusoid	$\cos(\omega_c t + \phi)$	$\frac{1}{2} [e^{j\phi} \delta(f - f_c) + e^{-j\phi} \delta(f + f_c)]$
Impulse	$\delta(t - t_d)$	$e^{-j\omega_d t_d}$
Sampling	$\sum_{k=-\infty}^{\infty} \delta(t - kT_s)$	$f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$
Signum	$\operatorname{sgn} t$	$1/j\pi f$
Step	$u(t)$	$\frac{1}{j2\pi f} + \frac{1}{2} \delta(f)$

# Trasformata di Fourier: proprietà in $f$ ed in $\omega$

$x(t)$	$X(\omega)$	$X(f)$
$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$	$X(f)e^{-j2\pi f t_0}$
$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$	$\frac{1}{ a } X\left(\frac{f}{a}\right)$
$X(t)$	$2\pi X(-\omega)$	$x(-f)$
$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$	$X(f - f_0)$
$\frac{d^n x(t)}{dt^n}$	$(j\omega)^n X(\omega)$	$(j2\pi f)^n X(f)$
$\int_{-\infty}^t x(\eta) d\eta$	$\frac{X(\omega)}{j\omega} + \pi X(0) \delta(\omega)$	$\frac{X(f)}{j2\pi f} + \frac{1}{2} X(0) \delta(f)$
$x(t) * y(t)$	$X(\omega)Y(\omega)$	$X(f)Y(f)$
$x(t)y(t)$	$\frac{1}{2\pi} X(\omega) * Y(\omega)$	$X(f) * Y(f)$
$x^*(t)$	$X^*(-\omega)$	$X^*(-f)$
$t^n x(t)$	$(-j)^{-n} \frac{d^n X(\omega)}{d\omega^n}$	$(-j2\pi)^{-n} \frac{d^n X(f)}{df^n}$

Notazione del Carlson:

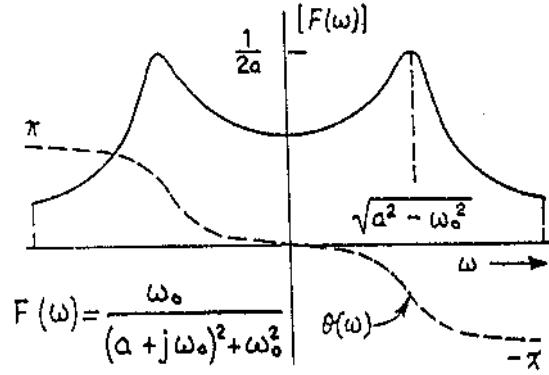
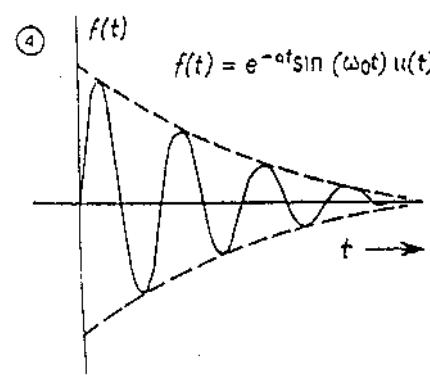
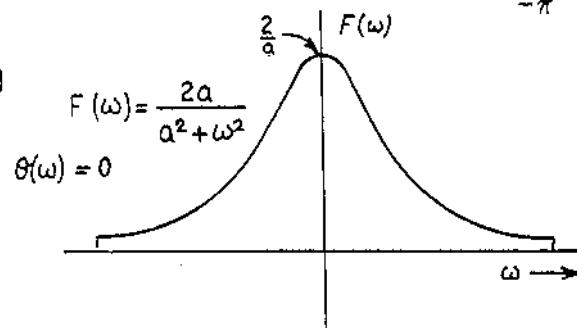
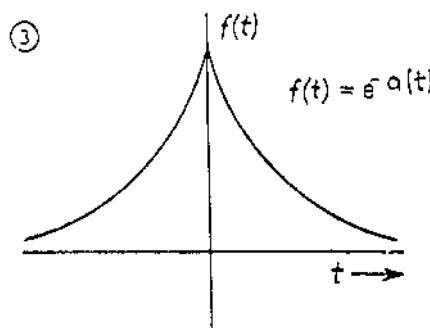
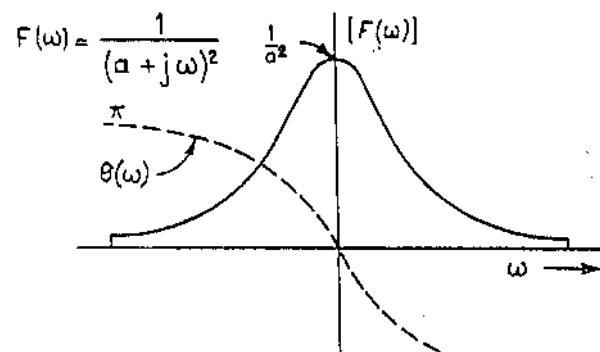
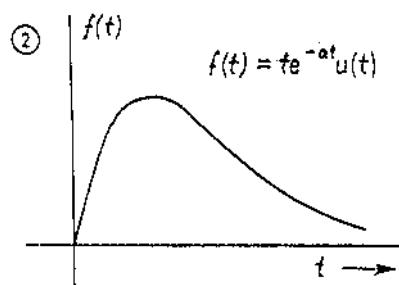
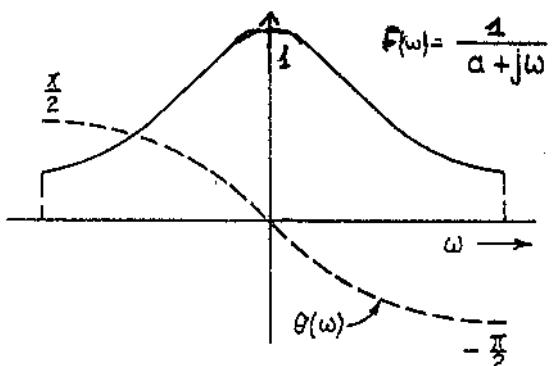
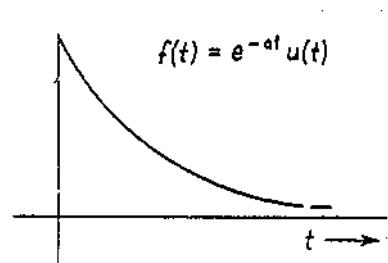


$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

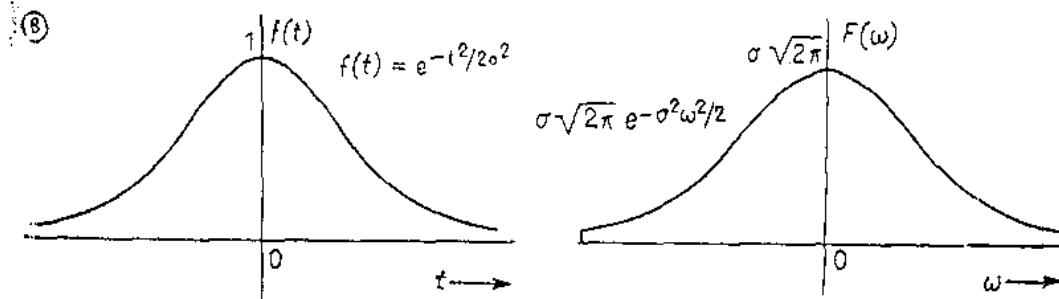
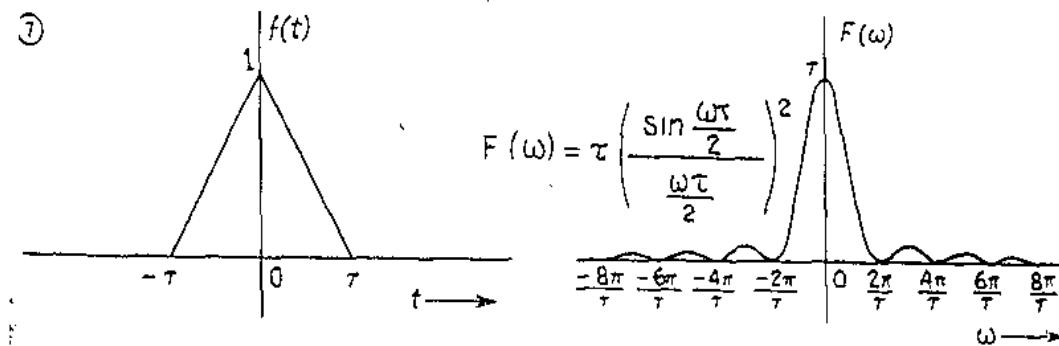
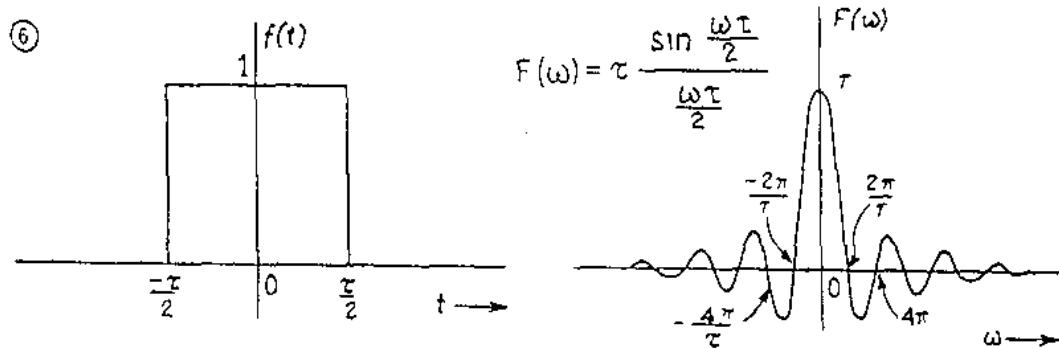
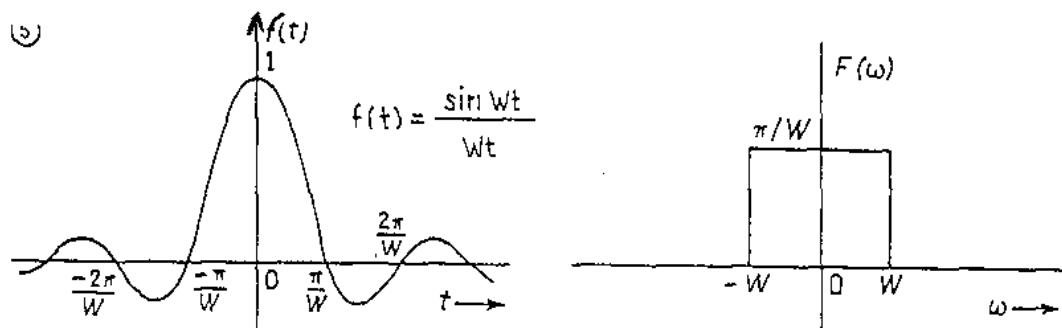
# Trasformate notevoli in $f$ ed in $\omega$

$\Pi\left(\frac{t}{\tau}\right)$	$\tau \text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$	$\tau \text{sinc}(f\tau)$
$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \text{sinc}^2\left(\frac{\omega\tau}{2\pi}\right)$	$\tau \text{sinc}^2(f\tau)$
1	$2\pi\delta(\omega)$	$\delta(f)$
$\cos(2\pi f_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$
$\sin(2\pi f_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$\frac{1}{2j}[\delta(f - f_0) - \delta(f + f_0)]$
$\delta(t)$	1	1
$\sum_k \delta(t - kT)$	$\frac{2\pi}{T} \sum_n \delta\left(\omega - n\frac{2\pi}{T}\right)$	$\frac{1}{T} \sum_n \delta\left(f - \frac{n}{T}\right)$
$\text{sinc}(Bt)$	$\frac{1}{B} \Pi\left(\frac{\omega}{2\pi B}\right)$	$\frac{1}{B} \Pi\left(\frac{f}{B}\right)$
$\text{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	$\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$
$e^{j(\omega_0 t + \varphi)}$	$2\pi\delta(\omega - \omega_0)e^{j\varphi}$	$\delta(f - f_0)e^{j\varphi}$
$e^{-b t }$	$\frac{2b}{b^2 + \omega^2}$	$\frac{2b}{b^2 + (2\pi f)^2}$
$e^{-bt}u(t)$	$\frac{1}{b + j\omega}$	$\frac{1}{b + j2\pi f}$

# Trasformate notevoli: grafici (1)

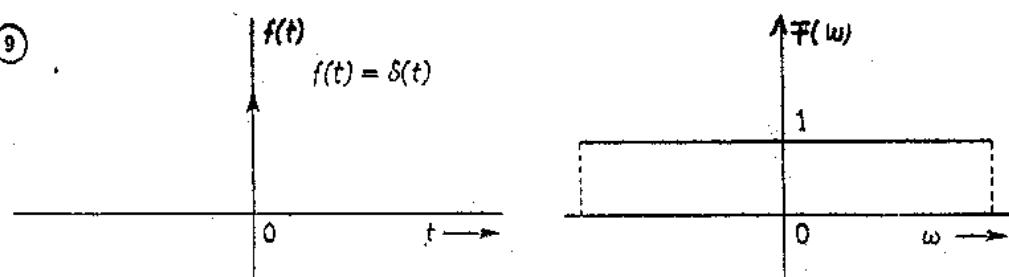


# Trasformate notevoli: grafici (2)

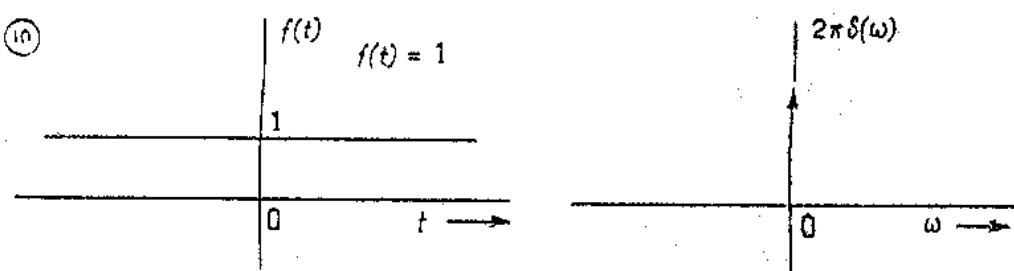


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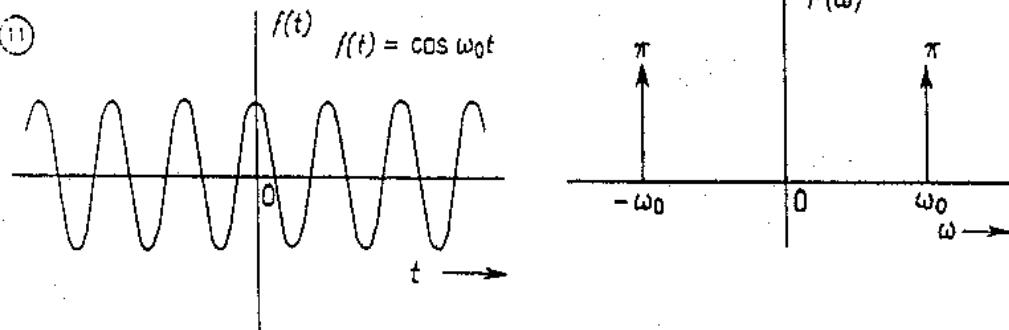
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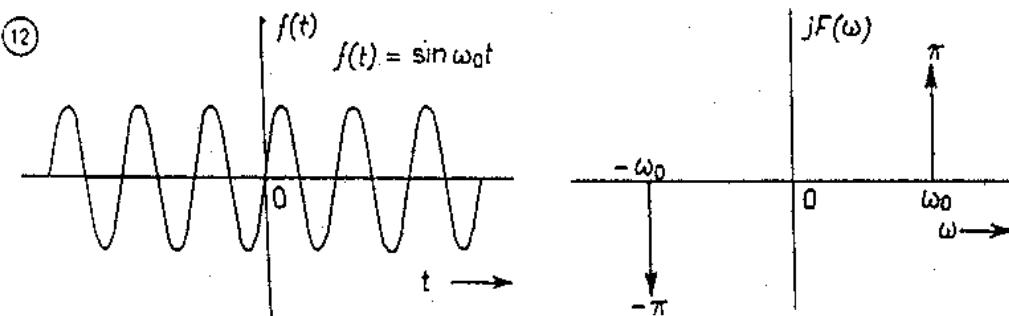
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# Trasformate notevoli: grafici (4)

